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MATHEMATICS

1. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_n$ are in arithmetic progression, whose common difference is an integer such that $a_1 = 1,\ a_n = 300$ and $n \in [15, 50]$, then (S_{n-4}, a_{n-4}) is

1) $(2491, 247)$ 2) $(2490, 248)$ 3) $(2590, 249)$ 4) $(248, 2490)$

Ans: 2

Sol: $a_n = a_1 + (n-1)d \Rightarrow 300 = 1 + (n-1)d$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{13 \times 23}{(n-1)} = \text{integer}$$

So $n - 1 = \pm 13, \pm 23, \pm 299, \pm 1$

$\Rightarrow n = 14, -12, 24, -22, 300, -298, 2, 0$

But $n \in [15, 50] \Rightarrow n = 24 \Rightarrow d = 13$

Hence $S_{n-4} = S_{20} = \frac{20}{2}\left[2(1) + (20-1)(13)\right] = 10[2 + 247] = 2490$

$a_{n-4} = a_{20} = a_1 + 19d$

$= 1 + 19 \times 13$

$= 1 + 247$

$= 248$

2. If $\lim\limits_{t \to x} \frac{x^2 f^2(t) - t^2 f^2(x)}{t - x} = 0$ and $f(1) = e$ then solution of $f(x) = 1$ is

1) $\frac{1}{e}$ 2) $\frac{1}{2e}$ 3) e 4) 2e

Ans: 1

Sol: $\lim\limits_{t \to x} \frac{x^2 f^2(t) - t^2 f^2(x)}{t - x} = 0$

Using L'Hospital

$$\lim_{t \to x} \frac{x^2 2f(t) f'(t) - 2t f^2(x)}{1} = 0$$

$x^2 2f(x) f'(x) - 2x f^2(x) = 0$

$2x f(x)\left[x f'(x) - f(x)\right] = 0$

$f(x) \neq 0$ so $x f'(x) = f(x)$

$x\frac{dy}{dx} = y$

$\frac{1}{y}dy = \frac{1}{x}dx$

Integer $l\,\mathrm{n}y = l\,\mathrm{n}x + l\,\mathrm{n}\,c$

$y = cx \Rightarrow f(x) = cx$

Now $f(1) = c = e$

So $f(x) = ex$

Now $f(x) = 1$

$ex = 1 \Rightarrow x = \frac{1}{e}$

3. Minimum value of $2^{\sin x} + 2^{\cos x}$ is

1) $2^{1-\frac{1}{\sqrt{2}}}$ 2) $2^{1+\frac{1}{\sqrt{2}}}$ 3) $2^{1+\sqrt{2}}$ 4) $2^{1-\sqrt{2}}$

Ans: 1

Sol: Using A.M.$\geq$ G.M.

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2.^{\sin x}.2^{\cos x}}$$

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq 2^{\frac{\sin x + \cos x}{2}} \quad \text{......................} \ (i)$$

Now $-\sqrt{2} \leq \sin x + \cos x = \sqrt{2}$

so $-\frac{1}{\sqrt{2}} \leq \frac{\sin x + \cos x}{2} \leq \frac{1}{\sqrt{2}}$

minimum value of $2^{\frac{\sin x - \cos x}{2}} = 2^{-\frac{1}{\sqrt{2}}}$

so by (i)

minimum value of $\frac{2^{\sin x} + 2^{\cos x}}{2} = 2^{-\frac{1}{\sqrt{2}}}$

minimum value of $2^{\sin x} + 2^{\cos x} = 2^{1}.2^{\frac{1}{\sqrt{2}}} = 2^{1-\frac{1}{\sqrt{2}}}$

4. The ratio of three consecutive terms in expansion of $(1+x)^{n+5}$ is $5:10:14$ then greatest coefficient is

1) 252 2) 462 3) 792 4) 320

Ans: 2

Sol: Let three consecutive term are T_r, T_{r+1}, T_{r+2}

Hence $\frac{T_r}{T_{r+1}} = \frac{5}{10}$ and $\frac{T_{r+1}}{T_{r+2}} = \frac{10}{14}$

$\frac{T_{r+1}}{T_r} = 2 \qquad \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{5}{7}$

$\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = 2 \qquad \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{7}{5}$

$n - r + 6 = 2r \qquad \frac{n-r+5}{r+1} = \frac{7}{5}$

$n - 3r + 6 = 0$ .... (i)

$5n - 5r + 25 = 7r + 7$

$5n - 12r + 18 = 0$ .... (ii)

Multiply equation (i) by 5

$5n - 15r + 30 = 0$

$5n - 12r + 18 = 0$

$- \qquad + \qquad -$

$-3r + 12 = 0 \Rightarrow r = 4,\ n = 6$

Hence greatest coefficient will be middle term = ${}^{n+5}C_5 = {}^{11}C_5 = 462$

5. There are 6 multiple choice questions in a paper each having 4 options of which only one is correct. In how many ways a person can solve exactly four correct, if he attempted all 6 questions.

1) 134 2) 135 3) 136 4) 137

Ans: 2

Sol: No. of ways of giving wrong answer = 3 required no. of ways $= {}^6C_4 (1)^4 \times (3)^2$

$= 15(9) = 135$

6.

Class	**0-10**	**10-20**	**20-30**
f	2	x	2

If variance of variable is 50 than x =

1) 5 2) 6 3) 4 4) 3

Ans: 3

Sol:

$\mathbf{x_i}$	**5**	**15**	**25**
$\mathbf{f_i}$	2	x	2

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{10+15x+50}{4+x}$$

$$= \frac{60+15x}{4+x} = 15$$

7. Two persons A and B play a game of throwing a pair of dice until one of them wins. A will win if sum of numbers of dice appear to be 6 and B will win, if sum is 7. What is the probability that A wins the game if A starts the game

1) $\frac{31}{61}$ 2) $\frac{30}{61}$ 3) $\frac{29}{61}$ 4) $\frac{32}{61}$

Ans: 2

Sol: $\text{sum } 6 \to (1,5), (5,1), (3,3), (2,4), (4,2)$

$\text{sum } 7 \to (1,6), (6,1), (5,2), (2,5), (3,4), (4,3)$

$$P(A \text{ wins}) = P(A) + P(\bar{A}).P(\bar{B}).P(A) + P(\bar{A})P(\bar{B}).P(\bar{A}).P(\bar{B}).P(A) + ...$$

This is infinite G.P. with common ratio $P(\bar{A}) \times P(\bar{B})$

Probability of A wins $= \dfrac{P(A)}{1-P(\bar{A})P(\bar{B})}$

$$= \frac{\frac{5}{36}}{1-\frac{31}{36}.\frac{30}{36}} = \frac{30}{61}$$

8. If ω is an imaginary cube roots of unity such that $(2+\omega)^2 = a + b\omega,\ a, b \in R$ then value of a + b is

1) 7 2) 6 3) 8 4) 5

Ans: 2

Sol; $(2+\omega)^2 = a + b\omega$

$4+\omega^2+4\omega = a+b\omega \quad \because 1+\omega^2 = -\omega$

$3+3\omega^2 = a+b\omega$

$(a-3)+\omega(b-3)=0$

$(a-3)+\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)(b-3)=0$

$(a-3)+\frac{1}{2}(b-3)+i\frac{\sqrt{3}}{2}(b-3)=0$

Compare real and imaginary part from both sides

$(a-3)+\frac{1}{2}(b-3)=0$ and $b-3=0 \Rightarrow b=3$ and $a=3$

Hence a +b = 6

9. Centre of a circle S passing through the intersection points of circles $x^2+y^2-6x=0$ & $x^2+y^2-4y=0$ lies on the line 2x – 3y + 12 = 0 then circle S passes through

1) (-3, 1) 2) (-4, -2) 3) (1, 2) 4) (-3, 6)

Ans: 4

Sol: By family of circle, passing through intersection of given circle will be member of $S_1+\lambda S_2=0$ family $(\lambda \neq 1)$

$\left(x^2+y^2-6x\right)+\lambda\left(x^2+y^2-4y\right)=0$

$(\lambda+1)x^2+(\lambda+1)y^2-6x-4\lambda y=0$

$x^2+y^2-\frac{6}{\lambda+1}x-\frac{4\lambda}{\lambda+1}y=0$

Centre $\left(\frac{3}{\lambda+1}, \frac{2\lambda}{\lambda+1}\right)$

Centre lies on $2x-3y+12=0$

$2\left(\frac{3}{\lambda+1}\right)-3\left(\frac{2\lambda}{\lambda+1}\right)+12=0$

$6\lambda+18=0$

$\lambda=-3$

Circle $x^2+y^2+3x-6y=0$

10. $$\int_{\pi/6}^{\pi/3} \tan^3 x.\sin^2 3x\left(2\sec^2 x.\sin^2 3x+3\tan x.\sin 6x\right)dx$$

1) $-\frac{1}{36}$ 2) $-\frac{1}{72}$ 3) $-\frac{1}{18}$ 4) $\frac{1}{36}$

Ans: 3

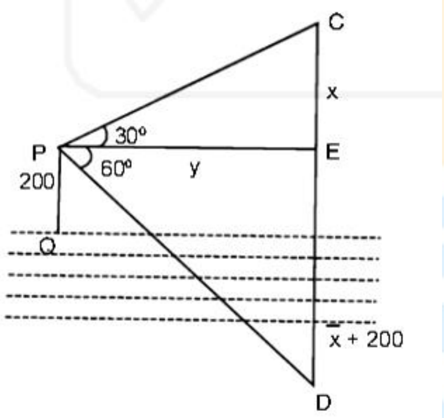
Sol: $$\int_{\pi/6}^{\pi/3}\left(\frac{\frac{d}{dx}\left(\tan^4 x\right)}{2}.\sin^4 3x+\tan^4 x.\frac{\frac{d}{dx}\left(\sin^4 3x\right)}{2}\right)=\frac{1}{2}\int_{\pi/6}^{\pi/3}\frac{d}{dx}\left(\tan^4 x.\sin^4 3x\right)dx$$

$$=\frac{1}{2}\left[\tan^4 x.\sin^4 3x\right]_{\pi/6}^{\pi/3}=\frac{1}{2}.\left[(3)^4\times O-\frac{1}{\left(\sqrt{3}\right)^4}\right]=-\frac{1}{2}\times\frac{1}{9}=-\frac{1}{18}]$$

11. From a pt 200 m above a lake, the angle of elevation of a cloud is $30°$ and the angle of depression of its reflection in lake is $60°$ then the distance of cloud from the point is

1) 400 m 2) $400\sqrt{2}$ m 3) $400\sqrt{3}$ m 4) 200 m

Ans: 1



Sol:

$$\text{tab}30° = \frac{x}{y} = \frac{1}{\sqrt{3}} \Rightarrow y = \sqrt{3}x \text{ and } \tan 60° = \frac{x+400}{y}$$

$$x + 400 = 3x$$

$$\sin 30° = \frac{200}{PC} \Rightarrow= PC = 400$$

12. The contrapositive of statement

"If f(x) is continuous at $x = a$ then f(x) is differentiable at $x = a$

1) If f(x) is continuous at $x = a$ then f(x) is not continuous at x =a

2) If f(x) is not differentiable at at $x = a$ then f(x) is not continuous at x =a

3) If f(x) is differentiable at at $x = a$ then f(x) is continuous at x =a

4) If f(x) is differentiable at at $x = a$ then f(x) is not continuous

Ans: 2

Sol: contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

13. If equation of directrix of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x=4$, then normal to the ellipse at point $(1,\beta),(\beta>0)$ passes through the point (where eccentricity of the ellipse is $\frac{1}{2}$)

1) $\left(1,\frac{3}{2}\right)$ 2) $\left(-1,\frac{3}{2}\right)$ 3) $(-1,-3)$ 4) $(3,-1)$

Ans: 1

Sol: $\frac{a}{e}=4 \Rightarrow a=4e \Rightarrow a=2$

$b^2=a^2\left(1-e^2\right)=3$

$(1,\beta)$ lies on $\frac{x^2}{4}+\frac{y^2}{3}=1 \Rightarrow \frac{1}{4}+\frac{\beta^2}{3}=1 \Rightarrow \beta^2=\frac{9}{4} \Rightarrow \beta=\frac{3}{2}(\because \beta>0)$

Normal at $(1,\beta) \Rightarrow \frac{a^2x}{1}-\frac{b^2y}{\beta}=a^2-b^2 \Rightarrow 4x-\frac{3y}{\beta}=1$

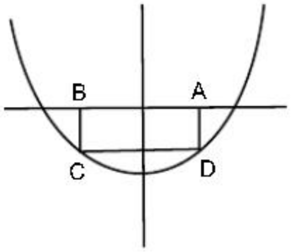
So equation of normal is $4x-2y=1$

14. If points A and B lie on x-axis and points C and D lie on the curve $y=x^2-1$ below the x-axis then maximum area of rectangle ABCD is

1) $\frac{4\sqrt{3}}{3}$ 2) $\frac{4\sqrt{3}}{9}$ 3) $\frac{4\sqrt{3}}{27}$ 4) $\frac{8\sqrt{3}}{9}$

Ans: 2

Sol:



$A(\alpha,0),\beta(-\alpha,0)$

$\Rightarrow D\left(\alpha,\alpha^2-1\right)$

Area $(ABCD)=(AB)(AD)$

$\Rightarrow S=(2\alpha)\left(1-\alpha^2\right)=2\alpha-2\alpha^3$

$\frac{ds}{d\alpha}=2-6\alpha^2=0\Rightarrow\alpha^2=\frac{1}{3}$

Area $=2\alpha-2\alpha^3=\frac{2}{\sqrt{3}}-\frac{2}{3\sqrt{3}}=\frac{4}{3\sqrt{3}}$

15. If α,β are roots of $x^2-x+2\lambda=0$ and α,γ are roots of $3x^2-10x+27\lambda=0$ then value of $\frac{\beta\gamma}{\lambda}$ is

1) 27 2) 18 3) 8 4) 15

Ans: 2

Sol: Given $3\alpha^2-10\alpha+27\lambda=0$ ...(i)

$3\alpha^2-3\alpha+6\lambda=0$ ...(ii)

Subtract $-7\alpha+21\lambda=0$

$3\lambda=\alpha$

By (ii) $9\lambda^2-3\lambda+2\lambda=0$

$\Rightarrow\lambda=0,\frac{1}{9}$

$\therefore$ given equation are $x^2-x+\frac{2}{9}=0$ and $3x^2-10x+3=0$

$\therefore\ \alpha=\frac{1}{3},\beta=\frac{2}{3},\alpha=\frac{1}{3},\gamma=3$

$\therefore\ \frac{\beta\gamma}{\lambda}=\frac{\frac{2}{3}.3}{\frac{1}{9}}=18$

16. If $\frac{dy}{dx}-\frac{y-3x}{\ell n\left(y-3x\right)}=3$, then

1) $\frac{\ell n\left(y-3x\right)}{2}=x+c$

2) $\frac{\ell n^{2}\left(y-3x\right)}{2}=x+c$

3) $\frac{\ell n\left(y-3x\right)}{2}=x^{2}+c$

4) $\frac{\ell n^{2}\left(y-3x\right)}{2}=x^{2}+c$

Ans: 2

Sol: $\frac{dy}{dx}=\frac{y-3x}{\ln\left(y-3x\right)}-3=0$

$$\frac{dy}{dx}-3=\frac{y-3x}{\ln\left(y-3x\right)}$$

$$\frac{d}{dx}\left(y-3x\right)=\frac{y-3x}{\ln\left(y-3x\right)}$$

$$\int\frac{\ell n\left(y-3x\right)}{\left(y-3x\right)}d\left(y-3x\right)=\int dx$$

Let $\ln\left(y-3x\right)=t$

$$\frac{1}{\left(y-3x\right)}d\left(y-3x\right)=dt$$

$$\int t\,dt=\int dt$$

$$\frac{t^{2}}{2}=x+c$$

$$\frac{\left(\ell n\left(y-3x\right)\right)^{2}}{2}=x+c$$

17. The distance of point (1, –2, –3) from plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

1) 7

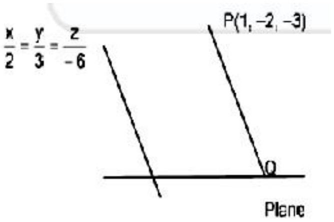
2) $\frac{1}{7}$

3) 1

4) 5

Ans: 4

Sol:



Equation PQ

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z+3}{-6}=\lambda$$

Let $Q=\left(2\lambda+1,3\lambda-2,-6\lambda-3\right)$

Q lies on $x-y+z=5$

$$\Rightarrow\left(2\lambda+1\right)-\left(3\lambda-2\right)+\left(-6\lambda-3\right)=5\Rightarrow\lambda=\frac{5}{7}\Rightarrow Q=\left(-\frac{3}{7},-\frac{29}{7},\right)$$

$$PQ=\sqrt{\left(1+\frac{3}{7}\right)^2+\left(-2+\frac{29}{7}\right)^2+\left(-3-\frac{9}{7}\right)^2}=\sqrt{\frac{100}{49}+\frac{225}{49}+\frac{900}{49}}=\sqrt{\frac{1225}{49}}=\frac{35}{7}=5$$

18. If $f\left(x\right)=\begin{cases}\frac{1}{2}\left(|x|-1\right), & \left(|x|>1\right)\\ \tan^{-1}x, & |x|\le 1\end{cases}$ then f(x) is

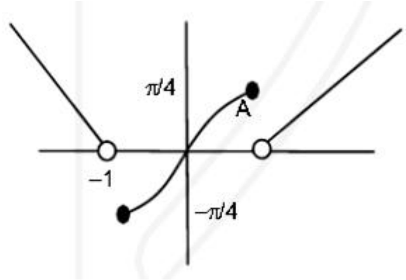
1) continuous for $x\in R-\{0\}$ 2) continuous for $x\in R-\{0,1,-1\}$

3) not continuous for $x\in\{-1,0,1\}$ 4) f(x) is continuous for $x\in R-\{1,-1\}$

Ans: 4

Sol: $\begin{cases}\frac{|x|-1}{2}, & |x|>1\\ \tan^{-1}x, & |x|\le 1\end{cases}$

Graph of f(x) is



f(x) is not continuous at $x=-1,1$

19. Suppose $X_1, X_2, ... X_{50}$ are 50 sets each having 10 elements and $Y_1, Y_2, ... Y_n$ are n sets each having 5 elements. Let $\overset{50}{\underset{i=1}{\cup}} X_i = \overset{n}{\underset{i=1}{\cup}} Y_i = Z$ and each element of Z belong to exactly 25 of X_i and exactly 6 of Y_i then value of n is

1) 20　　2) 22　　3) 24　　d) 26

Ans: 3

Sol: $\overset{50}{\underset{i=1}{\cup}} X_i = \overset{n}{\underset{i=1}{\cup}} Y_i = Z$

$$\therefore \frac{10\times 50}{25} = \frac{5n}{6} \Rightarrow n = 24$$

20. Let A is 3×3 matrix such that $AX_1 = B_1$, $AX_2 = B_2$. $AX_3 = B_3$

Where

$$x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix} \qquad x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix} \qquad x_2 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$B_1 = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad B_2 = \begin{bmatrix}0\\2\\0\end{bmatrix} \qquad B_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$$

Then find $|A|$

1) 0　　2) 1　　3) 2　　4) 3

Ans: 3

Sol: Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$Ax_1 = B_1 \Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = s$$

$a_1 + a_2 + a_3 = 1$

$b_1 + b_2 + b_3 = 0$

$c_1 + c_2 + c_3 = 0$

Similar $2a_2 + a_3 = 0$ and $a_3 = 0$

$2b_2 + b_3 = 2$ $\quad$ $b_3 = 0$

$2c_2 + c_3 = 0$ $\quad$ $c_3 = 2$

$\therefore a_2 = 0, b_2 = 1, c_2 = -1$

$a_1 = 1, b_2 = 1, c_2 = -1$

$a_1 = 1, b_1 = -1, c_1 = -1$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \therefore [A] = 2$$

21. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ then the value of $\left|\hat{i} \times \left(\vec{a} \times \hat{i}\right)\right|^2 + \left|\hat{j} \times \left(\vec{a} \times \hat{j}\right)\right|^2 + \left|\hat{k} \times \left(\vec{a} \times \hat{k}\right)\right|^2$ is

Ans: 18.00

Sol: $\hat{i} \times \left(\vec{a} \times \hat{i}\right) = \left(\hat{i}.\hat{i}\right)\vec{a} - \left(\vec{a}.\hat{i}\right)\hat{i} = y\hat{j} + z\hat{k}$

Similarly $\hat{j} \times \left(\vec{a} \times \hat{j}\right) = x\hat{i} + z\hat{k}$ and $\hat{k} \times \left(\hat{a} \times \hat{k}\right) = x\hat{i} + y\hat{k}$

$$\left|\hat{i} \times \left(a \times \hat{i}\right)\right|^2 + \left|\hat{j} \times \left(\vec{a} \times \hat{j}\right)\right|^2 + \left|\hat{k} \times \left(\vec{a} \times \hat{k}\right)\right|^2$$

$$\left|y\hat{j} + z\hat{k}\right|^2 + \left|x\hat{i} + z\hat{k}\right|^2 + \left|x\hat{i} + y\hat{j}\right|^2 = 2(9) = 18$$

22. $\int_0^n \{x\}\,dx, \int_0^n [x]\,dx$, and $10\left(n^2-n\right)$ are in geometric progression, where $\{x\}$ & $[x]$ represents fractional part function and greatest integral function respectively, find n if $n \in N$ and $n > 1$

Ans: 21.00

$$\int_0^n \{x\}\,dx = n\int_0^n x\,dx = n\left(\frac{x^2}{2}\right)_0 = \frac{n}{2} \text{ and } \int_0^n [x]\,dx = n\int_0^n \left(x-\{x\}\right)dx = \left(\frac{x^2}{2}\right)_0^n - \int_0^n \{x\}\,dx = \frac{n^2}{2} - \frac{n}{2}$$

now $\frac{n}{2}, \frac{n^2-n}{2}$ and $10\left(n^2-n\right)$ are in Geometric progression

$$=\left(\frac{n^2-n}{2}\right) = \frac{n}{2}.10\left(n^2-n\right) \Rightarrow \frac{n^2(n-1)^2}{4} = 5.n^2(n-1) \Rightarrow 20 \Rightarrow n = 21$$

23. PQ is a diameter of circle $x^2+y^2=4$. If perpendicular distance of P and Q from line $x+y=2$ are α and β respectively then maximum value of $\alpha\beta$ is

Ans: 2

Sol: Let $P\left(2\cos\theta, 2\sin\theta\right) \therefore Q\left(-2\cos\theta, -2\sin\theta\right)$

Given line $x+y-2=0$

$$\therefore \alpha = \frac{\left|2\cos\theta + 2\sin\theta - 2\right|}{\sqrt{2}}$$

$$\beta = \frac{\left|-2\cos\theta - 2\sin\theta - 2\right|}{\sqrt{2}}$$

$$\therefore \alpha\beta = \sqrt{2}\left(\cos\theta+\sin\theta-1\right).\sqrt{2}\left(\cos\theta+\sin\theta+1\right)$$

$$= 2\left(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1\right) = 2\sin\theta$$

$\therefore$ maximum $\alpha\beta = 2$